Casimir Effect in the Presence of Minimal Lengths

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Abstract

It is expected that the implementation of minimal length in quantum models leads to a consequent lowering of Planck's scale. In this paper, using the quantum model with minimal length of Kempf et al [3], we examine the effect of the minimal lenght on the Casimir force between parallel plates.

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1 Introduction

The construction of a quantized theory which incorporate gravity remains one of the priorities of theoretical physicists. Unfortunately all the attempts toward this goal fail. The reason is that the Planck scale $l_{\mathrm{p}} = 1.61605 \times 10^{-35}\,\mathrm{m}$, at which the effects of quantum gravity reveal themselves is so small, that these effects are neglected in experimentally accessible energies. Recently, to cure this problem, different scenarios have been proposed and all leading to a significant lowering of Planck's scale. Among them, models with large eXtra dimensions (LXD) [1], non commutative field theory models [2] and models with non zero minimal lengths [3]. In this paper we are interested in the later models based on generalized commutation relations $[\hat{x}_i, \hat{p}_j] = i\hbar \left[\left(1 + \beta \hat{p}^2\right) \delta_{ij} + \beta' \hat{p}_i \hat{p}_j \right]$. These commutations relations lead to a generalized uncertainty principle (GUP) which define non zero minimal lengths in position and/or momentum. A non zero minimal length in position has first appeared in the context of perturbative string theory [4]. One major feature of this finding is that the physics below such a scale becomes inaccessible and then define a natural cut-off which prevents from the usual UV divergencies. The other consequence of such GUP is the appearance of an intriguing UV/IR mixing, first noticed in the ADS/CFT correspondence [5]. Physically the UV/IR mixing means that we can probe short distances physics by long distances physics. We point that the UV/IR mixing is also a feature of non commutative quantum field theory [2, 6]. On the other hand some scenarios have been proposed where non zero minimal length is related to large eXtra dimensions [7], to the running coupling constant [8] and to the physics of black holes production [9].

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Recently the cosmological constant problem and the classical limit of the physics with minimal length have been investigated by the group of Virginia Tech [10, 11]. In [11] the value of the minimal length is so small that it seems meaningful. The size of the minimal length have been also extracted from the energy spectrum of the Coulomb potential [12, 13] and from the energy spectrum of electrons in a trap [14].

On the other hand the Casimir force has been calculated in a model incorporating one large eXtra dimension [15]. The comparison with available experimental data gives $R \lesssim 10nm$ where R is the size of the compactified eXtra dimension. Motivated by the fact that large eXtra dimensions and minimal lengths models aim to lower Planck's scale and can be related to each others we calculate in this paper, the effect of the presence of a minimal length on the Casimir force between parallel plates.

The rest of the paper is organized as follows. In section II, implementing the minimal length using standard methods of quantum mechanics we obtain generalized uncertainty principle (GUP), generalized plane waves and modified closure relations. In section III, we quantify the electromagnetic field and then following the standard recipe we calculate the Casimir force between to parallel plates. Section IV is left for concluding remarks.

2 Quantum mechanics with generalized Heisenberg relation

Following [3] we consider the following realization of the position and momentum operators

$$X_i = i\hbar[(1+\beta\mathbf{p}^2)\frac{\partial}{\partial p_i}], \qquad P_i = p_i, \quad . \tag{1}$$

where β is a small positive parameter. This representation leads to the following generalized commutators

$$[X_i, P_j] = i\hbar\delta_{ij}\left(1+\beta\mathbf{p}^2\right), \tag{2}$$

$$[X_i, X_j] = 2i\hbar\beta\left(P_iX_j - P_jX_i\right), \tag{3}$$

$$[P_i, P_j] = 0. \tag{4}$$

and the generalized uncertainty principle (GUP)

$$(\Delta X_i)\,(\Delta P_i) \geq \frac{\hbar}{2}\left[1+\beta(\Delta\mathbf{p})^2\right]. \tag{5}$$

The peculiarity of relation (5) is that it exhibits the UV/IR mixing phenomenon which allows to probe short distance physics (UV) from long distance one (IR). A minimization of (5) with respect to (ΔP_i) gives the following non zero minimal length

$$(\Delta X_i)_{min} = \hbar\sqrt{\beta}. \tag{6}$$

Eq.(6), like the UV/IR mixing, reveals the non local character of models based on Eqs.(1-3). Then we have not localized eigenfunctions in the $\mathbf{r}$-space. So, any eigenvalue problem can be solved by going to the momentum space.

In the following we derive necessary relations for our calculation taking in mind that we must recover the usual quantum mechanics in the limit $\beta \to 0$. First we assume that $\mathbf{R} \mid \mathbf{r} >= \mathbf{r} \mid \mathbf{r} >$ where the

vectors $\mid \mathbf{r}>$ represent maximally localized states. They are normalized states unlike the ones of ordinary quantum mechanics.
Using these maximally localized states we derive the following quasi-position eigenvectors

$$f_{\mathbf{p}}\left(\mathbf{r}\right)=\frac{1}{\sqrt[3]{2\pi\hbar}}\exp\left(-\frac{i\mathbf{r}}{\hbar\sqrt{\beta}}\arctan\mathbf{p}\sqrt{\beta}\right)\quad . \tag{7}$$

with the following generalized dispersion relation

$$\lambda\left(|\mathbf{p}|\right)=\frac{2\pi\hbar\sqrt{\beta}}{\arctan\left(\mid\mathbf{p}\mid\sqrt{\beta}\right)}. \tag{8}$$

The states given by (7) are far from being the well known plane waves. However in the limit $\beta\to 0$ we recover the usual planes waves of ordinary quantum mechanics.
Now assuming the usual closure relation for the maximally localized eigenstates $1=\int_{-\infty}^{+\infty}d\mathbf{r}'\mid\mathbf{r}><\mathbf{r}\mid$, we obtain

$$<\mathbf{p}'\mid\mathbf{p}>=\sqrt{\beta}\delta\left(\arctan\sqrt{\beta}\mathbf{p}-\arctan\sqrt{\beta}\mathbf{p}'\right). \tag{9}$$

Using the relation $\delta f(x)=\Sigma_i\frac{\delta(x-x_i)}{f'(x_i)}$, where x_i are the roots of $f(x)$, we finally get

$$<\mathbf{p}'\mid\mathbf{p}>=(1+\beta\mathbf{p}^2)^{\frac{1}{2}}(1+\beta\mathbf{p}'^2)^{\frac{1}{2}}\delta(\mathbf{p}-\mathbf{p}'). \tag{10}$$

From this equation we derive the modified completeness relation for the momentum eigenstates $\mid\mathbf{p}>$

$$\int\frac{d\mathbf{p}}{(1+\beta\mathbf{p}^2)}\mid\mathbf{p}><\mathbf{p}\mid=\mathbf{1}. \tag{11}$$

Here we observe a squeezing of the momentum space at high momentum. Let us end our calculations by showing that the states $\mid\mathbf{r}>$, like the coherent states, do not form an orthogonal set. Indeed we have

$$\begin{aligned}<\mathbf{r}\mid\mathbf{r}'>&=\int\frac{d\mathbf{p}}{(1+\beta\mathbf{p}^2)}f_{\mathbf{p}}\left(\mathbf{r}\right)f_{\mathbf{p}}^{*}\left(\mathbf{r}'\right)\\&=\int\frac{d\mathbf{p}}{(1+\beta\mathbf{p}^2)}\exp\left\{-\frac{i(\mathbf{r}-\mathbf{r}')}{\hbar\sqrt{\beta}}\arctan\sqrt{\beta}\mathbf{p}\right\}\\&=\frac{1}{\pi(\mathbf{r}-\mathbf{r}')}\sin\left(\frac{\pi(\mathbf{r}-\mathbf{r}')}{2\hbar\sqrt{\beta}}\right).\end{aligned} \tag{12}$$

The right hand is a well behaved function unlike the Dirac distribution of ordinary quantum mechanics. It is clear that the limit $\beta\longrightarrow 0$ restores the usual normalization $<\mathbf{r}\mid\mathbf{r}'>=\delta(\mathbf{r}-\mathbf{r}')$. In conclusion we have chosen to work with the normalization constant $1/\sqrt[3]{2\pi\hbar}$, while this choice renders the states given by Eq. (7) unphysicals, to reproduce in the limit $\beta\to 0$ the usual results of quantum mechanics.

3 Casimir effect

The most general solution of Maxwell equations in the presence of a minimal length in the Coulomb gauge for slowly moving particles is given by

$$\hat{\mathbf{A}}\left(\mathbf{r},t\right)=\sqrt{2\pi\hbar^3c\sqrt{\beta}}\int\frac{d\mathbf{p}}{(1+\beta\mathbf{p}^2)\sqrt{\arctan\sqrt{\beta}\mid\mathbf{p}\mid}}\sum_{\gamma=\pm1}\left[f_\gamma\left(\mathbf{p},\omega\right)\hat{a}_\gamma\left(\mathbf{p}\right)+f^{*}\left(\mathbf{p},\omega\right)\hat{a}_\gamma^{\dagger}\left(\mathbf{p}\right)\right] \tag{13}$$

where $f_\gamma(\mathbf{p},\omega)$ are generalized plane waves which can be obtained from Eq.(7)

$$f_\gamma(\mathbf{p},\omega) = \frac{\varepsilon_\gamma(\mathbf{p})}{\sqrt[3]{2\pi\hbar}} \exp\left(\frac{i}{\hbar\sqrt{\beta}}\left[\mathbf{r}\arctan\left(\mathbf{p}\sqrt{\beta}\right) - \hbar\omega\left(\mid\mathbf{p}\mid\right)t\right]\right), \tag{14}$$

with $\omega(\mid\mathbf{p}\mid)$ defined by the generalized dispersion relation (8) and $\varepsilon_\gamma(\mathbf{k})$ are the polarization vectors verifying

$$\varepsilon_\gamma(\mathbf{p})\,\varepsilon^*_{\gamma'}(\mathbf{p}) = \delta_{\gamma\gamma'}. \tag{15}$$

From (14) we derive the following normalization condition

$$\int d\mathbf{r} f^*_\gamma(\mathbf{p},\omega)\, i\overleftrightarrow{\partial}_0 f_{\gamma'}(\mathbf{p}',\omega') = \delta_{\gamma\gamma'}(1+\beta\mathbf{p}^2)^{\frac{1}{2}}(1+\beta\mathbf{p}'^2)^{\frac{1}{2}}\delta(\mathbf{p}-\mathbf{p}') \tag{16}$$

The creation and annihilation operators are non relativistic ones and, since the momentum operators are commuting, they satisfy the usual commutation relation,

$$\left[\hat{a}_\gamma(\mathbf{p}), \hat{a}^\dagger_{\gamma'}(\mathbf{p}')\right] = \delta_{\gamma\gamma'}\delta(\mathbf{p}-\mathbf{p}'). \tag{17}$$

This result, along with Eq.(12), can be used to derive a modified commutation relation between the fields

$$\left[A^i(\mathbf{r},t), E^j(\mathbf{r}',t)\right] = i\left(\delta^{ij} - \frac{\partial^i\partial^j}{\nabla^2}\right)\frac{\sin\left(\frac{\pi(\mathbf{r}-\mathbf{r}')}{2\hbar\sqrt{\beta}}\right)}{\pi(\mathbf{r}-\mathbf{r}')}. \tag{18}$$

Using the well known relation $\delta(x) = \lim_{\varepsilon\to 0}\frac{\sin(x/\varepsilon)}{\pi x}$ we obtain the usual commutation relation in the limit $\beta \to 0$.

Armed with this background, let us then attack the Casimir effect with square parallel plates of sides L. Then the electromagnetic field must satisfy boundary conditions. In our case we have from (14)

$$\kappa_3 = \frac{\hbar n\pi}{a}, \tag{19}$$

where a is the plates separation, $\kappa_3 = \frac{p_3}{\sqrt{\beta(\mathbf{q}^2+p_3^2)}}\arctan\left(\sqrt{\beta(\mathbf{q}^2+p_3^2)}\right)$ and $\mathbf{q}$ is the transverse momentum along the plates. In (19) we have a finite number of modes $n = 0, 1, 2, .., n_{\max} = \left[\frac{a}{2(\Delta x)_{\min}}\right]$ where $[\cdots]$ denotes the next smaller integer. Then the geometrical quantization given by (19) fulfills the requirement that in quantum models with a minimal length, Compton wavelength cannot take arbitrary values. Indeed we have $\lambda_{\min} = 4\hbar\sqrt{\beta}$.

Since β is a small parameter we have tried a series solution to the eight order in β. In the following we just show the following truncated solution

$$p_3(n) = \frac{\hbar n\pi}{a}\left[1 + \frac{\beta}{3}\left(\mathbf{q}^2 + \left(\frac{\hbar n\pi}{a}\right)^2\right) + \frac{\beta^2}{45}\left(2\left(\mathbf{q}\frac{\hbar n\pi}{a}\right)^2 - 4\mathbf{q}^4 + 12\left(\frac{\hbar n\pi}{a}\right)^4\right) + \cdots\right]. \tag{20}$$

In figure 1 we have plotted the modified wavelengths associated with momentums κ_3 and p_3 to the eight order in β for $\beta = 0.01$ and $\hbar = a = q = 1$. For large n the wavelength associated with κ_3 tends asymptotically to $\lambda_{\min}$ while the one associated with p_3 tends to zero faster than the wavelength of the usual theory. A similar behavior has been obtained in [16] using generalized dispersion relations.

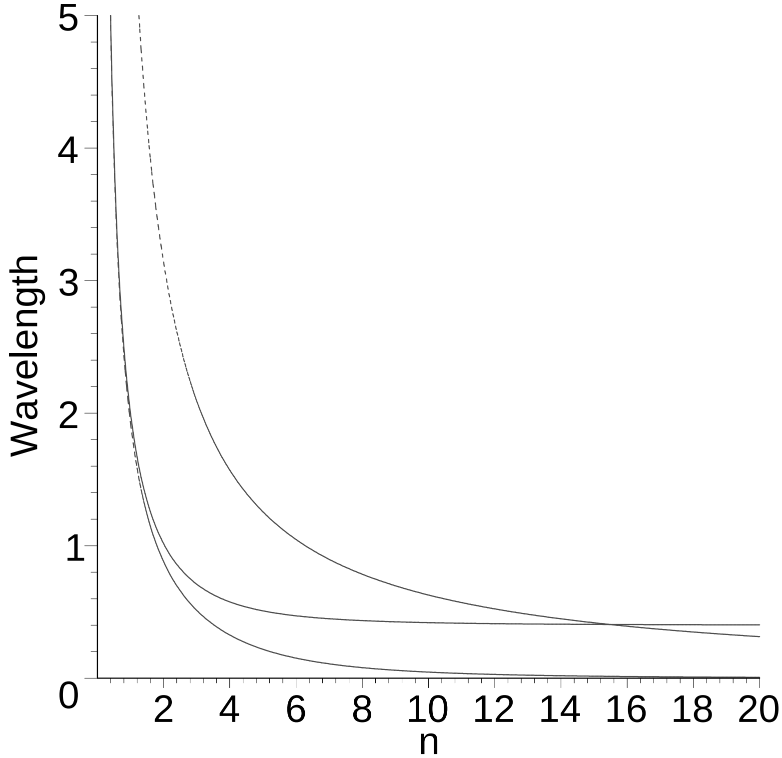

Figure 1: Plot of Compton wavelengths associated with the momentums κ_3 (solide), p_3 (dot) for β=0.01 and the usual one (dash-dot) versus the quantum number n.

The potential vector in the presence of the plates is then given by

$$\hat{\mathbf{A}}_a(\mathbf{r},t) = \sqrt{2\pi\hbar^3 c\sqrt{\beta}}\frac{\hbar\pi}{a}\sum_{\substack{n=-n_{\max}\\ \gamma=\pm1}}^{n_{\max}}\int\frac{d\mathbf{q}}{\left(1+\beta\mathbf{p}^2\left(a\right)\right)\sqrt{\arctan\sqrt{\beta}\mid\mathbf{p}\left(a\right)\mid}}$$
$$\times\left\{f_\gamma\left(\mathbf{p}\left(a\right),\omega\right)\hat{a}_\gamma\left(\mathbf{p}\left(a\right)\right)+f^*\left(\mathbf{p}\left(a\right),\omega\right)\hat{a}_\gamma^\dagger\left(\mathbf{p}\left(a\right)\right)\right\}, \tag{21}$$

where

$$\mathbf{p}\left(a\right)=\mathbf{q}+\mathbf{p}_3\left(n\right). \tag{22}$$

The commutation relation between the creation and annihilation operators is then affected by the solution (20). For our purpose it suffice to we use the following approximation

$$\left[\hat{a}_\gamma\left(\mathbf{p}\left(a\right)\right),\hat{a}_{\gamma'}^\dagger\left(\mathbf{p}'\left(a\right)\right)\right]\simeq\frac{a}{\pi\hbar}\delta_{nn'}\delta_{\gamma\gamma'}\delta(\mathbf{q}-\mathbf{q}')+O\left(\beta\right). \tag{23}$$

The energy shift resulting from the presence of the plates is defined by the relation

$$\begin{aligned}\Delta E &=< 0\mid\left(\hat{H}(a)-\hat{H}\right)\mid 0>\\ &=\frac{1}{8\pi}\int d\mathbf{r}<0\mid\left\{\left(\partial_0\hat{\mathbf{A}}_a\right)^2-\hat{\mathbf{A}}_a\Delta\hat{\mathbf{A}}_a+\left(\partial_0\hat{\mathbf{A}}\right)^2-\hat{\mathbf{A}}\Delta\hat{\mathbf{A}}\right\}\mid 0>.\end{aligned} \tag{24}$$

Performing the standard calculation we get

$$\Delta E=\frac{cL^2}{8\pi\hbar^2\beta^{\frac{1}{2}}}\int d\mathbf{q}\left\{\sum_{n=-n_{\max}}^{n_{\max}}\frac{\arctan\sqrt{\beta\left(\mathbf{q}^2+p_3^2(n)\right)}}{1+\beta\left(\mathbf{q}^2+p_3^2(n)\right)}-\int_{-\nu_{\max}}^{\nu_{\max}}d\nu\frac{\arctan\sqrt{\beta\left(\mathbf{q}^2+p_3^2(\nu)\right)}}{1+\beta\left(\mathbf{q}^2+p_3^2(\nu)\right)}\right\}. \tag{25}$$

From this expression it is easily seen that terms proportional to $\beta^{n\geq1}$ in $p_3(n)$ and the omitted terms in the commutation relation (23) will give negligible contributions proportional to $\beta^{n\geq2}$.
Exchanging sums and integrals and defining the following quantity

$$G(\nu)=\frac{1}{\sqrt{\beta}}\int_0^\infty dx\frac{\arctan\sqrt{\beta\left(p_3^2\left(\nu\right)+x\right)}}{1+\beta\left(p_3^2\left(\nu\right)+x\right)}, \tag{26}$$

the energy shift per unit area $\Delta\mathcal{E}=\frac{\Delta E}{L^2}$ is given by

$$\Delta\mathcal{E}=\frac{c}{4\pi\hbar^2}\left\{\sum_{n=0}^{n_{\max}}G(n)-\int_0^{\nu_{\max}}d\nu G(\nu)-\frac{1}{2}G(0)\right\}. \tag{27}$$

With the aid of the variable $\rho=\frac{1}{\sqrt{\beta}}\arctan\sqrt{\beta\left(x+p_3^2\left(\nu\right)\right)}$, the function $G\left(\nu\right)$ is simply given by

$$G(\nu)=\frac{2}{\sqrt{\beta}}\int_{\frac{1}{\sqrt{\beta}}\arctan p_3(\nu)\sqrt{\beta}}^{\frac{\pi}{2\sqrt{\beta}}}\tan\left(\sqrt{\beta}\rho\right)\rho d\rho. \tag{28}$$

Using the following expansion [18]

$$t\tan t=\sum_{k=1}^{\infty}\frac{2^{2k}\left(2^{2k}-1\right)B_k}{(2k)!}t^{2k},\quad |t|<\frac{\pi}{2} \tag{29}$$

and performing the integral over ρ we obtain

$$G(\nu)=\sum_{k=1}^{\infty}\frac{\beta^{k-1}2^{2k+1}\left(2^{2k}-1\right)B_k}{(2k+1)(2k)!}\left[\left(\frac{\pi}{2\sqrt{\beta}}\right)^{2k+1}-\left(\frac{1}{\sqrt{\beta}}\arctan p_3(\nu)\sqrt{\beta}\right)^{2k+1}\right] \tag{30}$$

where B_k are Bernoulli's numbers given by $B_1=\frac{1}{6}$, $B_2=\frac{1}{30}, B_3=\frac{1}{42},\cdots$ [17].

It is important to note here that we have not introduced any cut-off as is the case in the ordinary Casimir effect. The cut-off $\frac{1}{\sqrt{\beta}}$ is implemented naturally in Eq.(28). In Eq.(30) the contributions for $n>n_{\max}$ are negligeables compared to the ones for $n\leq n_{\max}$ since $\frac{1}{\sqrt{\beta}}\arctan p_3(\nu)\sqrt{\beta}$ tends asymptotically to $\frac{\pi}{2\sqrt{\beta}}$ for $n>n_{\max}$. For the rest of the calculation the first term is irrelevant for our purpose and we ignore it. Then we can extend the summation over n and ν in Eq.(27) from 0 to $+\infty$. Thus

$$\Delta\mathcal{E}=\frac{c}{4\pi\hbar^2}\left\{\sum_{n=0}^{\infty}G(n)-\int_0^{\infty}d\nu G(\nu)-\frac{1}{2}G(0)\right\}. \tag{31}$$

At this stage we can use Euler formula [17]

$$\int_0^{\infty}f(x)dx=\sum_{n=0}^{\infty}f(n)-\frac{f(0)}{2}+\sum_{m=1}^{\infty}\frac{B_{2m}}{(2m)!}f^{(2m-1)}(0) \tag{32}$$

to obtain

$$\Delta\mathcal{E}=-\frac{c}{4\pi\hbar^2}\sum_{m=1}^{\infty}\frac{B_{2m}}{(2m)!}G^{(2m-1)}(0) \tag{33}$$

where B_{2m} are Bernoulli numbers and $G^{(l)}(0)$ are derivatives of $G(\nu)$ at $\nu=0$.

Using the expression of $p_3(\nu)$ given by (20) in (30) we obtain to a first order expansion in β (Recall that the commutation relations are valid to first order in β)

$$G(\nu)=-4B_1\left(\frac{\hbar\pi\nu}{a}\right)^3+4\beta\left[\frac{B_1}{3}+B_2\right]\left(\frac{\hbar\pi\nu}{a}\right)^5. \tag{34}$$

Using $B_1=\frac{1}{6}$, $B_2=\frac{1}{30}$ we finally obtain

$$G(\nu)=-\frac{2}{3}\left(\frac{\hbar\nu\pi}{a}\right)^3+\frac{48}{135}\beta\left(\frac{\hbar\nu\pi}{a}\right)^5. \tag{35}$$

Then from Eq.(33) we have

$$\Delta\mathcal{E}=-\frac{c}{4\pi\hbar^2}\left[\frac{B_4}{4!}G^{(3)}(0)+\frac{B_6}{6!}G^{(5)}(0)\right]. \tag{36}$$

Evaluating the derivatives at $\nu=0$ and using $B_4=\frac{1}{30}$, $B_6=\frac{691}{2730}$ we obtain

$$\Delta\mathcal{E}=\hbar c\left\{\frac{\pi^2}{720a^3}-\beta\frac{691}{284275}\frac{\hbar^2\pi^4}{a^5}\right\}. \tag{37}$$

The force per unit surface $\mathcal{F} = \frac{\partial}{\partial a}\Delta\mathcal{E}$ generated by this energy is given by

$$\mathcal{F} = -\hbar c \left\{ \frac{1}{240}\frac{\pi^2}{a^4} - \beta \frac{691}{36855}\frac{\hbar^2\pi^4}{a^6} \right\}. \tag{38}$$

It is clear from this result, that for a fixed separation of the plates, the Casimir force in the presence of a minimal length may be attractive of repulsive depending on the value of the minimal length $(\Delta x)_{\min} = \hbar\sqrt{\beta}$.

The first term in Eq.(38) is the standard attractive Casimir force [19] which, alone, is a source of instability. Indeed the two plates systems can collapse to a one plate system.The second term which is the correction arising from the presence of the minimal length is the repulsive contribution to Casimir force and therefore provides the desired stability of the two plates systems. This is important for the construction of consistent Kaluza-Klein theories. The same results have been obtained by [20] for the Casimir effect in κ-deformed theory and by [16] for a particular implementation of the minimal length.

The condition for a quantum stability of the two plates systems gives the following constraint

$$\frac{(\Delta x)_{\min}}{a} \sim 0.15. \tag{39}$$

Using the experimentally accessible plates separations, which are of order 100 nm [22], we obtain

$$\hbar\sqrt{\beta} \sim 15 \text{ nm}. \tag{40}$$

However for the force to remains attractive, as is usually observed, we have the condition $\frac{\hbar\sqrt{\beta}}{a} \lesssim 0.15$.

Figure 2 illustrate the variation of Casimir force for different values of the minimal length. It is clear that this force becomes repulsive for $\frac{(\Delta x)_{\min}}{a} > 0.15$. Let us point that in the plot a is always greater than $(\Delta x)_{\min}$ because the Casimir force for plates separation below the minimal length is meaningful since the space below this scale is fuzzy and then experimentally inaccessible.

Before ending this section we note that the Casimir force in the presence of one compactified eXtra dimension lies below the standard Casimir force [15], while in the of a minimal length it lies above. Therefore we can conclude that the effects of the minimal length and the eXtra dimensions are opposites. This is expected from the beginning since the minimal length squeeze the momentum space at high momentum and then the natural cut-off of the model suppress the contributions of such momentum. Finally our treatment along with the work in [16] contradict the one in [21] where the Casimir force in the presence of a minimal length has been found to be a discontinuous function of the plates separation, a result essentially due to an appropriate geometric quantization between the plates.

4 Conclusion

In this paper we considered the effect of minimal length on the Casimir force between parallel plates. We shown that the minimal length acts like a natural cut-off which suppress the contribution of unwanted high momentum. Using the accessible plates separation used for an experimental calculation of Casimir force we found an upper bound for the minimal length of the same order of the size of one compactified eXtra dimension. However this bound is already excluded from high precision mesurments and collider

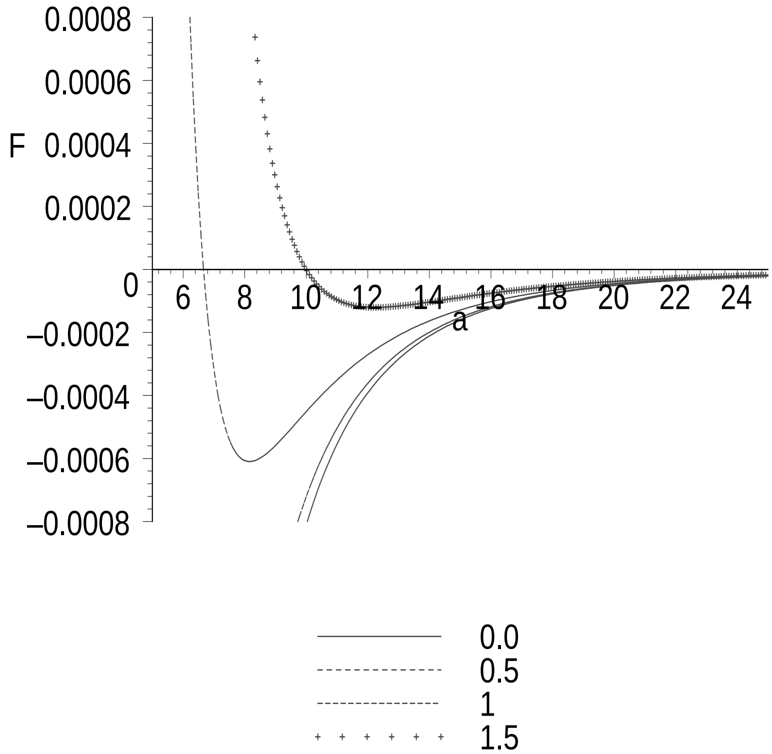

Figure 2: Plot of Casimir Force F [eV/nm^3] versus the plates separation a [nm] for different values of the minimal length.

experiments [7] and then we recover the usual attractive character of Casimir force. The Casimir force in the presence of minimal length in the context of a model with one eXtra dimension is under investigation and will be published elsewhere.

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